PROBLEM RELATIVE TO THE MOVE OF THE KNIGHT AT THE GAME OF CHESS.

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"The Knight being placed on any given square of the chess board, it is required, at sixty-three successive moves, to cause it to move over the remaining sixty-three squares."

This curious question is perhaps well known to most of those who are familiar with the game of chess. It was considered of sufficient importance in the time of Euler for him to furnish an extended article on it to the St. Petersburg Academy.

I propose to show that the Knight may be placed on any square of the chess board and, by sixty-three or sixty-four moves, may land on any other designated square of the board, moving in its course over every square on the chess board.

There is a great variety of courses which the Knight may take, starting from any particular square, and move over all the remaining sixty-three squares at sixty-three successive moves. If we can once obtain what may be termed a reëntering course, such as is shown in figures 1 and 2, the last move being but one move from the starting point, it is plain that we may from this course obtain as many courses as we please.

Suppose, for instance, we wish to have the Knight end his course on No. 28, Fig. 1. All we have to do is to reverse the numbers, putting 64 in place of 28, 63 for 29, 62 for 30, 61 for 31, &c., until we arrive at 64, which will be 28; then, afterward, from 27 down to 1 the moves will be unchanged.

In the same manner, if we wished the Knight to end his course on No. 56, reversing the numbers to 64, then from 55 to 1 the course is unchanged. Pursuing the same course, we may, in a very few transformations, have the Knight end his course on any one of the squares represented by even Nos.

FIG. 1.

52	7	24	31	50	35	12	33
25	30	51	8	11	32	49	36
6	53	10	23	48	13	34	59
29	26	47	54	9	60	37	14
46	5	28	41	22	15	58	61
27	2	45	16	55	42	21	38
4	17	64	43	40	19	62	57
1	44	3	18	63	56	39	20

FIG. 2.

45	32	9	18	47	30	7	20
10	17	46	31	8	19	48	29
33	44	55	58	51	62	21	6
16	11	52	61	54	57	28	49
43	34	59	56	63	50	5	22
12	15	64	53	60	25	38	27
35	42	13	2	37	40	23	4
14	1	36	41	24	3	26	39

It will be observed that, diagonally across the board in either direction, the rows of figures are alternately odd and even, and, from the nature of the moves of the Knight, it would be impossible for it, in sixty-three moves, to commence and land on any square in the odd rows. When a new course is found in the way proposed another new course may be obtained from this one, and, commencing with Fig. 1, in the corner, the Knight may be made to move successively over all the remaining sixty-three squares and end his journey on any one of the squares in the alternate rows represented by even numbers. In Fig. 2, the Knight is supposed to commence on another square. Here, as in the 1st figure, each alternate row, diagonally, has odd and even numbers and the Knight can be made to end his journey on any square represented by even numbers, the same as in Fig. 1, in sixty-three moves.

It may further be observed that, in either Fig. 1 or Fig. 2, if we wish to have the Knight *end* on *any square on the board*, regardless of where the starting point is, it can be done, by one transformation, in this manner:

Suppose in Fig. 2 we want the Knight to end his course on No. 29. Instead of 29, 28, 27, &c., write 64, 63, 62, &c., down to No. 1 on the board. The whole course will be 29 to 1, written 64 to 36, and 64 to 30 written 35 to 1. The starting point of this new course will be 30, ending at 29. And in this way courses may be found, without number, beginning and ending within one move of each other; each course being a reëntering course.

The Knight, from the nature of his moves, cannot, at sixty-three moves, begin and end his course on any of the squares represented by odd or even numbers; if he commences on an odd row he must end on an even row, and vice versa. So that by placing the Knight on any square represented by an odd number, he may finish his course ending on either of the thirty-two even numbers, and vice versa; and again these courses, in each case, may be varied in many ways. The Knight may thus in reëntering courses begin or end on any particular square that may be named. The recreation is a pleasant one to practice, and upon once getting started is quite easy to perform.

Euler's article was mainly to show how to obtain by trial and changes what I have here termed a reëntering course.

CORRESPONDENCE.—EDITOR OF ANALYST: Problem 258, published in ANALYST, No. 2, Vol. VI, probably admits of no purely mathematical solution, as several numbers of the ANALYST have since appeared and no solution has been given.

The only attempt at an exact determination of the arc required, that I know of, is by Prof. Phillips of the Polytechnic school at Paris; the result